exo 7

transfo Abel $\left.\begin{array}{l} (a_n)_{n\geq 0} \searrow \text{ cv vers } 0 \ (\mathbb{R}) \\ \left(\sum_{k=0}^{n} b_k\right)_{n\geq 0} \text{ bornée (F en dim finie)} \end{array}\right\} \Rightarrow \sum a_n b_n \text{ cv}$

méthode = transfo. Abel

$$\sum_{n=0}^{N} a_n b_n = \sum_{n=1}^{N} a_n (S_n - S_{n-1}) + a_0 \overbrace{b_0}^{S_0}$$

$$= \sum_{n=0}^{N} a_n S_n - \sum_{n=0}^{N-1} a_{n+1} S_n \qquad \swarrow n \to n+1$$

$$= \sum_{n=0}^{N} (a_n - a_{n+1}) S_n + a_{N+1} S_N$$

or $|(a_n - a_{n+1}) S_n| \leq \underbrace{M}_{S \text{ bornée}} \underbrace{(a_n - a_{n+1})}_{\geq 0}$ dc $\sum$ abs cv par comp.

($\sum (a_n - a_{n+1})$ cv car télescopique et (a_n) cv), dc cv

$a_{N+1} S_N \longrightarrow 0$ car $a_N \to 0$ S_N bornée dc $\sum a_n b_n$ cv

rqe $\displaystyle\sum_{n=p}^{p+N} a_n b_n = \underbrace{\sum_{n=p}^{p+N} (a_n - a_{n+1}) S_n}_{} + a_{p+1+N} S_{p+N} + a_p S_{p+1}$

$$= \sum_{n=p}^{p+N} a_n S_n - \sum_{n=p+1}^{p+1+N} a_n S_{n-1} = \sum_{n=p}^{p+N} a_n b_n - a_{p+1+N} S_{p+N} + a_p S_{p+1}$$

$$\left|\sum_{n=p}^{\infty} a_n b_n\right| \leq M \underbrace{\sum_{n=p}^{\infty} (a_n - a_{n+1})}_{a_p} + 2 a_p M \leq 3 M a_p$$

↪ permet de montrer la cvu

≥ 0