Chapitre(s) 0 : Packs de démarrage
Feuille d'exercices - correction partielle

Exercice 4 :

Soit $x \in \mathbb{R}$. Réduire les expressions suivantes (on suppose que les dénominateurs ne s'annulent pas) :

$A = \dfrac{1}{x-7} + \dfrac{3}{2x-1}$. $\quad B = \dfrac{3}{(x-1)(x+2)} + 5$. $\quad C = \dfrac{3(x+2)}{x^2} \times \dfrac{x(x+1)}{2(x+3)} \div \dfrac{2x+2}{x}$

$$A = \frac{5x-22}{(x-7)(2x-1)}.$$
$$B = \frac{5x^2+5x-7}{(x-1)(x+2)}.$$
$$C = \frac{3(x+2)}{4(x+3)}$$

petite subtilité dans C : on exploite le fait que $2x+2 = 2(x+1)$ ce qui donne une simplification rapide...

Exercice 5 :

Soient $x, y \in \mathbb{R}^*$. Réduire les expressions suivantes :

1. $A = x^{-1} \times \dfrac{x^7}{x^4}$.
2. $B = \dfrac{x^2}{(x^{-2})^3}$.
3. $C = \dfrac{x^{-2}y^3}{(xy^{-1})^3}$.

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1. $A = x^2$.
2. $B = x^8$.
3. $C = \dfrac{y^6}{x^5}$.

Exercice 6 :

Réduire les expressions suivantes :

1. $A = 9^{n+2} - 9^{n+1} + 2 \times 3^{2n}$.
2. $B = \dfrac{2}{4^n} - 7 \times 2^{-2n-1} + 5 \times \left(\dfrac{1}{2}\right)^{2n}$.
3. $C = 3^{2n}(-1)^n - (-9)^n$.

L'astuce est d'utiliser le fait que $a^{n+p} = a^n a^p$ et $a^{np} = (a^n)^p$, afin de factoriser le plus possible les expressions.

1.
$$\begin{aligned} A &= 9^n 9^2 - 9^n \times 9 + 2 \times 9^n \\ &= 9^n(81 - 9 + 2) \\ A &= 74 \times 9^n \end{aligned}$$

2.
$$\begin{aligned} B &= \frac{2}{4^n} - 7\frac{1}{2^{2n+1}} + 5\frac{1}{2^{2n}} \\ &= 2\frac{1}{4^n} - 7\frac{1}{4n}\frac{1}{2} + 5\frac{1}{4^n} \\ &= \frac{1}{4^n}(2 - \frac{7}{2} + 5) \\ B &= \frac{7}{2} \times \frac{1}{4^n} = \frac{7}{2 \times 4^n} \end{aligned}$$

3.
$$\begin{aligned} C &= (3^2)^n(-1)^n - (-1 \times 9)^n \\ &= 9^n(-1)^n - (-1)^n 9^n \\ C &= 0 \end{aligned}$$

Exercice 7 :

Résoudre les systèmes linéaires suivants en utilisant la méthode du pivot de Gauss :

1. $\left\{\begin{array}{rcl} 2x-3y & = & 1 \\ x-y & = & -2 \end{array}\right.$

2. $\left\{\begin{array}{rcl} 3x-3y & = & 1 \\ 7x-y & = & 2 \end{array}\right.$

3. $\left\{\begin{array}{rcl} 2x-3y & = & 3x \\ x-y & = & y \end{array}\right.$

4. $\left\{\begin{array}{rcl} 2x+4y-4z & = & -8 \\ 3x+9y-6z & = & 9 \\ 4x+17y-11z & = & 41 \end{array}\right.$

5. $\left\{\begin{array}{rrrl} x & -y & & =0 \\ -3x & +4y & +2z & =5 \\ -2x & +3y & +z & =3 \end{array}\right.$

6. $\left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ x & -y & +z & =2 \\ 4x & +y & +z & =3 \end{array}\right.$

7. $\left\{\begin{array}{rrrl} 2x & +y & +z & =1 \\ x & -y & -z & =2 \\ 4x & -y & -z & =3 \end{array}\right.$

8. $\left\{\begin{array}{rrrl} x & +y & +2z & =4 \\ & y & +2z & =1 \\ 2x & -y & +z & =8 \end{array}\right.$

9. $\left\{\begin{array}{rcl} -x+6y-z & = & 7 \\ 2x-5y+3z & = & 2 \end{array}\right.$

5.

$$\left\{\begin{array}{rrrl} x & -y & & =0 \\ -3x & +4y & +2z & =5 \\ -2x & +3y & +z & =3 \end{array}\right. \underset{\substack{L_2 \leftarrow L_2+3L_1 \\ L_3 \leftarrow L_3+2L_2}}{\Longleftrightarrow} \left\{\begin{array}{rrrl} x & -y & & =0 \\ & y & +2z & =5 \\ & y & +z & =3 \end{array}\right.$$

$$\underset{L_3 \leftarrow L_2-L_3}{\Longleftrightarrow} \left\{\begin{array}{rrrl} x & -y & & =0 \\ & y & +2z & =5 \\ & & z & =2 \end{array}\right.$$

$$\Longleftrightarrow \left\{\begin{array}{rl} x & =1 \\ y & =1 \\ z & =2 \end{array}\right.$$

6.

$$\left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ x & -y & +z & =2 \\ 4x & +y & +z & =3 \end{array}\right. \underset{\substack{L_2 \leftarrow L_1-2L_2 \\ L_3 \leftarrow 2L_1-L_3}}{\Longleftrightarrow} \left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ & 3y & -3z & =-3 \\ & y & -3z & =-1 \end{array}\right.$$

$$\underset{L_3 \leftarrow L_2-3L_3}{\Longleftrightarrow} \left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ & 3y & -3z & =-3 \\ & & 6z & =0 \end{array}\right.$$

$$\Longleftrightarrow \left\{\begin{array}{rl} x & =1 \\ y & =-1 \\ z & =0 \end{array}\right.$$

7.

$$\left\{\begin{array}{rrrl} 2x & +y & +z & =1 \\ x & -y & -z & =2 \\ 4x & -y & -z & =3 \end{array}\right. \underset{\substack{L_2 \leftarrow L_1-2L_2 \\ L_3 \leftarrow 2L_1-L_3}}{\Longleftrightarrow} \left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ & 3y & +3z & =-3 \\ & 3y & +3z & =-1 \end{array}\right.$$

$$\Longleftrightarrow \left\{\begin{array}{rrrl} 2x & +y & -z & =1 \\ & 3y & +3z & =-3 \\ & 0 & =-2 & \end{array}\right.$$

Système incompatible.

8.

$$\left\{\begin{array}{rrrl} x & +y & +2z & =4 \\ & y & +2z & =1 \\ 2x & -y & +z & =8 \end{array}\right. \underset{L_3 \leftarrow 2L_1-L_3}{\Longleftrightarrow} \left\{\begin{array}{rrrl} x & +y & +2z & =4 \\ & y & +2z & =1 \\ & 3y & +3z & =0 \end{array}\right.$$

$$\underset{L_3 \leftarrow 3L_2-L_3}{\Longleftrightarrow} \left\{\begin{array}{rrrl} x & +y & +2z & =4 \\ & y & +2z & =1 \\ & & +3z & =3 \end{array}\right.$$

$$\Longleftrightarrow \left\{\begin{array}{rl} x & =3 \\ y & =-1 \\ z & =1 \end{array}\right.$$

9. Ce système aura nécessairement une inconnue secondaire, puisqu'il n'y a que deux équations pour 3 inconnues.

$$\left\{\begin{array}{rcl} -x+6y-z & = & 7 \\ 2x-5y+3z & = & 2 \end{array}\right. \underset{L_2 \leftarrow 2L_1+L_2}{\Longleftrightarrow} \left\{\begin{array}{rcl} -x+6y-z & = & 7 \\ 7y+z & = & 16 \end{array}\right.$$

$$\Longleftrightarrow \left\{\begin{array}{rcl} x & = & -23-13y \\ z & = & 16-7y \end{array}\right.$$