

Corrigé des exercices pour la rentrée.

Rappels lycée**Exercice 1** Calculs à faire très rapidement :

- $(2,5 \times 6,7) + (2,5 \times 3,3) = 2,5 \times (6,7 + 3,3) = 2,5 \times 10 = 25$
- $(9 \times (4 + \frac{2}{3})) + (9 \times (5 + \frac{1}{3})) = 9 \times [(4 + \frac{2}{3}) + (5 + \frac{1}{3})] = 9 \times 10 = 90$
- $(5,2 \times 5,6) + (5,2 \times 4,4) = 5,2 \times (5,6 + 4,4) = 5,2 \times 10 = 52$
- $(12 \times (3 + \frac{3}{4})) + (12 \times (6 + \frac{1}{4})) = 12 \times [(3 + \frac{3}{4}) + (6 + \frac{1}{4})] = 12 \times 10 = 120$
- $(0,54 \times 80,5) + (0,54 \times 19,5) = 0,54 \times (80,5 + 19,5) = 0,54 \times 100 = 54$
- $(11 \times (9 + \frac{6}{5})) - (11 \times (9 - \frac{1}{5})) = 11 \times [(9 + \frac{6}{5}) - (9 - \frac{1}{5})] = 11 \times 2 = 22$
- $(8,6 \times 71,9) + (8,6 \times 28,1) = 8,6 \times (71,9 + 28,1) = 8,6 \times 100 = 860$
- $(6 \times (97 + \frac{7}{4})) + (6 \times (1 + \frac{1}{4})) = 6 \times [(97 + \frac{7}{4}) + (1 + \frac{1}{4})] = 6 \times 100 = 600$
- $(1,63 \times 51,5) + (1,63 \times 48,5) = 1,63 \times (51,5 + 48,5) = 1,63 \times 100 = 163$
- $8 \times (2 + \frac{5}{3}) + 8 \times (4 + \frac{7}{3}) = 8 \times (2 + \frac{5}{3} + 4 + \frac{7}{3}) = 8 \times (6 + \frac{12}{3}) = 80$
- $7,9^2 - 2,1^2 = (7,9 - 2,1)(7,9 + 2,1) = 5,8 \times 10 = 58$

Exercice 2 Réduire au même dénominateur :

- $\frac{1}{7} - \frac{2}{5} = \frac{5}{35} - \frac{14}{35} = \frac{5-14}{35} = -\frac{9}{35}$
- $\frac{2}{3} + \frac{18}{7} = \frac{14}{21} + \frac{54}{21} = \frac{14+54}{21} = \frac{68}{21}$
- $\frac{3}{21} - \frac{4}{9} = \frac{1}{7} - \frac{4}{9} = \frac{9}{63} - \frac{28}{63} = \frac{9-28}{63} = -\frac{19}{63}$
- $\frac{7}{25} + \frac{3}{12} - \frac{1}{20} = \frac{7}{25} + \frac{1}{4} - \frac{1}{20} = \frac{28}{100} + \frac{25}{100} - \frac{5}{100} = \frac{48}{100} = \frac{12}{25}$
- $\frac{1}{30} - \frac{5}{42} + \frac{7}{15} = \frac{1}{30} - \frac{5}{42} = \frac{1}{2} - \frac{5}{42} = \frac{21-5}{42} = \frac{8}{21}$

Exercice 3 Quelques calculs élémentaires à faire rapidement :

- $\frac{8}{6} - \frac{9}{4} = \frac{4}{3} - \frac{9}{4} = \frac{16}{12} - \frac{27}{12} = \frac{16-27}{12} = \frac{-11}{12}$
- $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right) \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right) = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
- $(-1) - (-2) + 3 - (-1) = -1 + 2 + 3 + 1 = 5$
- $1 - \frac{1}{3} + \frac{1}{1 + \frac{1}{3}} = 1 - \frac{1}{3} + \frac{1}{\frac{4}{3}} = 1 - \frac{1}{3} + \frac{3}{4} = \frac{12}{12} - \frac{4}{12} + \frac{9}{12} = \frac{12-4+9}{12} = \frac{17}{12}$
- $\frac{3}{7} \times \left(\frac{11}{3} + \frac{16}{5}\right) - \left(\frac{3}{7} + \frac{11}{3}\right) \times \frac{16}{5} = \frac{11}{3} \times \left(\frac{3}{7} - \frac{16}{5}\right) = \frac{11}{3} \times \frac{15-112}{35} = \frac{11 \times 98}{3 \times 35} = \frac{11 \times 7 \times 14}{3 \times 35} = \frac{154}{15}$
- $\frac{\frac{3}{4} - \frac{5}{3}}{\frac{1}{3} + \frac{4}{5}} = \frac{\frac{9}{12} - \frac{20}{12}}{\frac{5+12}{15}} = \frac{\frac{-11}{12}}{\frac{17}{15}} = -\frac{11}{12} \times \frac{15}{17} = -\frac{165}{204} = -\frac{55}{68}$
- $\frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{3} - \sqrt{2}} = \frac{(\sqrt{3} - \sqrt{2}) + (\sqrt{3} + \sqrt{2})}{\sqrt{3}^2 - \sqrt{2}^2} = \frac{2\sqrt{3}}{3-2} = 2\sqrt{3}$

8. $\frac{\sqrt{20} - 3\sqrt{5}}{\sqrt{8}\sqrt{10} - 5} = \frac{2\sqrt{5} - 3\sqrt{5}}{4\sqrt{5} - 5} = \frac{-\sqrt{5}}{4\sqrt{5} - 5} \cdot \frac{4\sqrt{5} + 5}{4\sqrt{5} + 5} = \frac{-\sqrt{5}(4\sqrt{5} + 5)}{(4\sqrt{5})^2 - 5^2} = \frac{-4 \cdot 5 + 5\sqrt{5}}{5 \times 16 - 5^2} = \frac{-4 + \sqrt{5}}{16 - 5} = -\frac{4 + \sqrt{5}}{11}$
9. $(\sqrt{3 + \sqrt{5}} + \sqrt{3 - \sqrt{5}})^2 = (a + b)^2 = a^2 + b^2 + 2ab = (3 + \sqrt{5}) + (3 - \sqrt{5}) + 2\sqrt{(3 + \sqrt{5})(3 - \sqrt{5})} = 6 + 2\sqrt{9 - 5} = 6 + 4 = 10$
10. $\left(1 + \sqrt{\frac{31}{3}}\right)^3 + \left(1 - \sqrt{\frac{31}{3}}\right)^3 = 2 + 2 \times 3 \left(\frac{31}{3}\right) = 2 + 62 = 64$

Exercice 4 Quelques développements élémentaires :

1. $(a - b)^3 = (a - b)(a - b)(a - b) = (a^2 - 2ab + b^2)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$
2. $(3a + b^2 + c)^2 = (3a)^2 + (b^2)^2 + c^2 + 2(3a)(b^2) + 2(3a)(c) + 2(b^2)(c) = 9a^2 + b^4 + c^2 + 6ab^2 + 6ac + 2b^2c$
3. $(a + b)^2 + (a - b)^2 = (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2) = 2a^2 + 2b^2$
4. $\left(a + \frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \left(\frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$
5. $((a + b)^2)^2 = (a^2 + 2ab + b^2)^2 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
6. $\left(a - \frac{1}{a}\right)^3 = a^3 - 3a^2 \cdot \frac{1}{a} + 3a \cdot \frac{1}{a^2} - \frac{1}{a^3} = a^3 - 3a + \frac{3}{a} - \frac{1}{a^3}$
7. $(b + c)^4 = (b + c)^2(b + c)^2 = (b^2 + 2bc + c^2)(b^2 + 2bc + c^2) = b^4 + 4b^3c + 6b^2c^2 + 4bc^3 + c^4$
8. $\left(a + \frac{1}{a}\right)^4 = \left(a^2 + 2 + \frac{1}{a^2}\right)^2 = a^4 + 4a^2 + 4 + \frac{1}{a^4} + 4 + 2a^2 = a^4 + 4a^2 + 4 + \frac{1}{a^4}$
9. $(a - b\sqrt{2})^2(a + b\sqrt{2})^2 = ((a - b\sqrt{2})(a + b\sqrt{2}))^2 = (a^2 - 2b)^2 = a^4 - 4a^2b + 4b^2$
10. $\left(\frac{a}{b} + \frac{b}{a}\right)^4 = \left(\frac{a^2 + b^2}{ab}\right)^4 = \frac{(a^2 + b^2)^4}{a^4b^4}$

Exercice 5 Quelques simplifications élémentaires :

1. $2 + \frac{2}{1 + \sqrt{3}} = 2 + \frac{2(1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})} = 2 + \frac{2(1 - \sqrt{3})}{1 - 3} = 2 - (1 - \sqrt{3}) = 3 - \sqrt{3}$
2. $1 + \frac{1}{1 + \frac{1}{1 + \sqrt{3}}} = 1 + \frac{1 + \sqrt{3}}{2 + \sqrt{3}} = 1 + \frac{(1 + \sqrt{3})(2 - \sqrt{3})}{4 - 3} = \sqrt{3}$
3. $1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \sqrt{3}}}}} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{2(1 - \sqrt{3})}{1 - 3}}} = 1 + \frac{1}{1 + \frac{1}{1 + \sqrt{3}}} = 1 + \frac{1 + \sqrt{3}}{2 + \sqrt{3}} = 1 + \frac{(1 + \sqrt{3})(2 - \sqrt{3})}{4 - 3} = \sqrt{3}$
4. $(a - 3)(b + c) - (ac + 2b) = (ab + ac - 3b - 3c - ac - 2b) = ab - 5b - 3c$
5. $(a - b)(a - 2c) + (b - c)(a - 2) = (a^2 - 2ac - ab + 2bc) + (ab - 2b - ac + 2c) = a^2 - 3ac + 2bc - 2b + 2c$
6. $(b^2 - a)^2 - a(2b^2 - a) = (b^4 - 2ab^2 + a^2) - (2ab^2 - a^2) = b^4 - 4ab^2 + 2a^2 = (b^2 - 2a)^2 - 2a^2 = (b^2 - (2 + \sqrt{2})a)(b^2 - (2 - \sqrt{2})a)$
7. $c(1 - a)(1 - b) + b(1 - a)(1 - c) + b(1 - a)(1 - c) = (1 - a)(c(1 - b) + 2b(1 - c)) = (1 - a)(2b + c - 3bc)$
8. $a^2c + (a - b)(b - c)(a + b) = a^2c + (a^2 - b^2)(b - c) = a^2c + a^2b - a^2c - b^3 + b^2c = b(a^2 - b^2 + bc)$
9. $(a - b + c)^2 - (-a - b + c)^2 + (a + b + c)^2 = [(a - b + c) - (-a - b + c)][(a - b + c) + (-a - b + c)] + (a^2 + 2ab + b^2 + 2ac + 2bc + c^2) = (2a)(-2b + 2c) + a^2 + 2ab + b^2 + 2ac + 2bc + c^2 = a^2 + b^2 + c^2 - 2ab + 6ac + 2bc$
10. $(3a + 2)^2 - (4a + b)(2a - 1) = (9a^2 + 12a + 4) - (8a^2 - 4a + 2ab - b) = a^2 + 16a + 4 - 2ab + b$
11. $-3 + a(1 + a(2 + b(-1 + a))) = -3 + a(1 + a(2 - b + ab)) = -3 + a + a^2(2 - b + ab) = -3 + a + 2a^2 - a^2b + a^3b$

Exercice 6 (Factorisations)

1. $2a^2 - 2ba = 2a(a - b)$

2. $4x^4 - 16y^4 = 4(x^4 - 4y^4) = 4(x^2 - 2y^2)(x^2 + 2y^2) = 4(x - \sqrt{2}y)(x + \sqrt{2}y)(x^2 + 2y^2)$

3. $16x^4 + 256 - 128x^2 = 16(x^4 - 8x^2 + 16) = 16(x^2 - 4)^2 = 16(x - 2)^2(x + 2)^2$

4. $-49x^3 - 9xy^2 + 42x^2y = -7x(7x^2 + 6xy + 9y^2)$

5. $2x^3 - 12x^2 - 54x = 2x(x^2 - 6x - 27) = 2x(x + 3)(x - 9)$

6. $-48x^3 + 48x^2 - 12x = -12x(4x^2 - 4x + 1) = -12x(2x - 1)^2$

7. $\frac{1}{4}x^3 + \frac{1}{9}xy^2 + \frac{1}{3}x^2y = \frac{1}{36}x(9x^2 + 4y^2 + 12xy) = \frac{x(3x + 2y)^2}{36}$

8. $-4x^9y + 4x^4y^6 - x^8y + x^3y^6 = yx^3(-4x^6 + 4xy^5 - x^5 + y^5) = yx^3(4x(-x^5 + y^5) - x^5 + y^5) = yx^3(y^5 - x^5)(4x + 1) = yx^3(4x + 1)(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

9. $7a^4 + 28a - 14a^3b - 56b = 7(a^4 - 2a^3b + 4a - 8b) = 7(a^3(a - 2b) + 4(a - 2b)) = 7(a - 2b)(a^3 + 4)$

10. $8x^2y - 4x - 6xy^2 + 3y = 2xy(4x - 3y) - 4x + 3y = (4x - 3y)(2xy - 1)$

11. $15ax + 6ay - 5bx - 2by = (3a - b)(5x + 2y)$

12. $20a^3b^2 + 12a^4b - 10a^2b - 6a^3 = 2a^2(10ab^2 + 6a^2b - 5b - 3a) = 2a^2(2ab(5b + 3a) - 5b - 3a) = 2a^2(5b + 3a)(2ab - 1)$

13. $3a^2x - 4a^2y^2 - 3bx + 4by^2 = (3a^2 - 3b)x - (4a^2 - 4b)y^2 = (3x - 4y^2)(a^2 - b)$

Exercice 7 Factorisez et déterminez le signe des expressions suivantes :

1. $x^3 - 2x^2 - 11x + 12 = (x - 3)(x^2 + x - 4) = (x - 3)(x - 2)(x + 2)$

2. $x^5 + x^4 + 6x^3 + 6x^2 + 9x + 9 = (x^2 + 3)(x^3 + x + 3)$

3. $x \ln(x) - x - 2 \ln(x) + 2 = (x - 2)(\ln(x) - 1)$

4. $x + 8 - \frac{16}{x - 7} = \frac{(x - 7)(x - 7) + 16}{x - 7} = \frac{(x - 7)^2 + 16}{x - 7}$

5. $xe^x + 3e^x - 2x - 6 = (e^x - 2)(x + 3)$

Exercice 8 (Réduction au même dénominateur) Voici les réductions :

1.

$$\frac{1}{x+1} - \frac{2}{x^2-1} + \frac{7}{x-1} = \frac{1}{x+1} - \frac{2}{(x-1)(x+1)} + \frac{7}{x-1} = \frac{x-1-2+7(x+1)}{(x+1)(x-1)} = \frac{8x+4}{(x+1)(x-1)}$$

2.

$$\begin{aligned} \frac{x-2}{x^2-5x+6} + \frac{x+1}{x-2} &= \frac{x-2}{(x-2)(x-3)} + \frac{x+1}{x-2} \\ &= \frac{(x+1)(x-3) + (x-2)}{(x-2)(x-3)} = \frac{x^2-2x-3+x-2}{(x-2)(x-3)} = \frac{x^2-x-5}{(x-2)(x-3)} \end{aligned}$$

3.

$$\begin{aligned} \frac{7}{a^2-b^2} + \frac{2}{a^2-ab} - \frac{5}{ab-b^2} &= \frac{7}{(a-b)(a+b)} + \frac{2}{a(a-b)} - \frac{5}{b(a-b)} \\ &= \frac{7ab+2b(a+b)-5a(a+b)}{ab(a-b)(a+b)} = \frac{4ab+2b^2-5a^2}{ab(a^2-b^2)} \end{aligned}$$

4.

$$\frac{a-b}{a^2-b^2} + \frac{a+b}{3a^2+6ab+3b^2} - \frac{ab}{a^2b+ab^2} = \frac{1}{a+b} + \frac{1}{3(a+b)} - \frac{1}{a+b} = \frac{1}{3(a+b)}$$

5.

$$\frac{1}{9a^2 - 9b^2} + \frac{1}{9(a+b)} = \frac{1}{9(a-b)(a+b)} + \frac{1}{9(a+b)} = \frac{1+a-b}{9(a-b)(a+b)}$$

6.

$$\frac{x}{2x-y} + \frac{y}{y^2 + 4x^2 - 4xy} = \frac{x}{2x-y} + \frac{y}{(y-2x)^2} = \frac{x(y-2x)+y}{(y-2x)^2} = \frac{xy-2x^2+y}{(y-2x)^2}$$

7.

$$\frac{2x+7y}{(x-y)(2x+3y)} - \frac{4}{(x-y)(3x-y)} = \frac{(2x+7y)(3x-y) - 4(2x+3y)}{(x-y)(2x+3y)(3x-y)} = \frac{6x^2+19xy-7y^2-8x-12y}{(x-y)(2x+3y)(3x-y)}$$

8.

$$\begin{aligned} \frac{4ab}{(a^2x+b)(x-a)} - \frac{a-b}{(4a+7)(a^2x+b)} + \frac{1}{(a^2x+b)(x-y)} \\ = \frac{4ab(4a+7)(x-y) - (a-b)(x-a)(x-y) + (x-a)(4a+7)}{(a^2x+b)(x-a)(4a+7)(x-y)} \end{aligned}$$

$$9. \frac{1}{2x} + \frac{1}{2x+2} = \frac{1}{2x} + \frac{1}{2(x+1)} = \frac{(x+1)+x}{2x(x+1)} = \frac{2x+1}{2x(x+1)}$$

10.

$$\begin{aligned} \frac{1}{2n-2} - \frac{1}{n} + \frac{1}{2n+2} &= \frac{1}{2(n-1)} - \frac{1}{n} + \frac{1}{2(n+1)} \\ &= \frac{1}{2(n-1)} - \frac{1}{n} + \frac{1}{2(n+1)} = \frac{n(n+1)}{2n(n-1)(n+1)} - \frac{2(n-1)(n+1)}{2n(n-1)(n+1)} + \frac{n(n-1)}{2n(n-1)(n+1)} \\ &= \frac{n(n+1) - 2(n^2-1) + n(n-1)}{2n(n-1)(n+1)} \\ &= \frac{n^2+n-2n^2+2+n^2-n}{2n(n-1)(n+1)} = \frac{2}{2n(n-1)(n+1)} = \frac{1}{n(n^2-1)} \end{aligned}$$

$$11. \frac{\frac{x}{x-y} - \frac{y}{x+y}}{\frac{y}{x-y} + \frac{x}{x+y}} = \frac{\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}}{\frac{y(x+y)+x(x-y)}{(x-y)(x+y)}} = 1$$

12.

$$\frac{2x-3}{x+1} + \frac{3}{x-1} - \frac{2x^2}{x^2-1} = \frac{(2x-3)(x-1) + 3(x+1) - 2x^2}{(x+1)(x-1)} = \frac{2(3-2x)}{(x+1)(x-1)}$$

$$13. \frac{1}{x - \frac{1}{3 + \frac{x-2}{5-x}}} = \frac{1}{x - \frac{5-x}{13-2x}} = \frac{1}{\frac{x(13-2x)-(5-x)}{13-2x}} = \frac{13-2x}{-2x^2+14x-5} = \frac{2x-13}{2x^2-14x+5}$$