

Prélude 3-4:

- EX1: 1) $\cos$ est pair donc $\cos(-\frac{\pi}{3}) = \cos(\frac{\pi}{3}) = \frac{1}{2}$
 2) $\cos$ est 2π -périodique donc $\cos(\frac{5\pi}{3}) = \cos(\frac{5\pi}{3} - 2\pi) = \cos(-\frac{\pi}{3}) = \frac{1}{2}$
 3) $\cos(-\frac{7\pi}{6}) = \cos(2\pi - \frac{7\pi}{6}) = \cos(\frac{5\pi}{6}) = \cos(\pi - \frac{\pi}{6}) = -\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$
 4) $\cos(\frac{2\pi}{3}) = \cos(\pi - \frac{\pi}{3}) = -\cos(\frac{\pi}{3}) = -\frac{1}{2}$
 5) $\sin$ est impair donc $\sin(-\frac{5\pi}{2}) = -\sin(\frac{5\pi}{2}) = -\sin(2\pi + \frac{\pi}{2}) = -\sin(\frac{\pi}{2}) = -1$
 6) $\sin(-\frac{31\pi}{6}) = -\sin(\frac{31\pi}{6}) = -\sin(5\pi + \frac{\pi}{6}) = -\sin(\frac{\pi}{6}) = -\frac{1}{2}$

EX2: $\cos(at+bt) = \cos(a + (b+c))$
 $= \cos a \cos(b+c) - \sin a \sin(b+c)$
 $= \cos a (\cos b \cos c - \sin b \sin c) - \sin a (\sin b \cos c + \cos b \sin c)$
 $= \cos a \cos b \cos c - \cos a \sin b \sin c - \sin a \sin b \cos c - \sin a \cos b \sin c$

EX3: 1) $\cos^2(a) = 1 - \sin^2(a) = \frac{21}{25}$ dc $\cos(a) = \pm \frac{\sqrt{21}}{5}$
 0 < a < $\frac{\pi}{2}$ ou $\cos$ est > 0 dc $|\cos(a)| = \frac{\sqrt{21}}{5}$

puis de $\tilde{m}$, $|\cos(b)| = \frac{4}{5}$

puis $\cos(a+b) = \cos a \cos b - \sin a \sin b = \frac{-4\sqrt{21} - 6}{25}$
 $\cos(a-b) = \cos a \cos b + \sin a \sin b = \frac{-4\sqrt{21} + 6}{25}$
 $\sin(a+b) = \sin a \cos b + \cos a \sin b = \frac{-8 + 3\sqrt{21}}{25}$
 $\sin(a-b) = \sin a \cos b - \cos a \sin b = \frac{-8 - 3\sqrt{21}}{25}$

2) $\sin(a) = \frac{\sqrt{3}}{2}$, $\cos(b) = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$

$\cos(a+b) = \frac{2\sqrt{2} + \sqrt{3}}{6}$, $\cos(a-b) = \frac{2\sqrt{2} - \sqrt{3}}{6}$

$\sin(a+b) = \frac{-2\sqrt{5} + 1}{6}$, $\sin(a-b) = \frac{-2\sqrt{5} - 1}{6}$

EX4:

- 1) $x \in D$ (i) $f(x)$ bien définie
 (ii) $\cos x \neq 0$ et $\sin(x) \neq 0$
 (iii) $x \in]0, \frac{\pi}{2}[\cup]\frac{3\pi}{2}, 2\pi[$

2) $\forall x \in D$, $f(x) = \frac{3\sin x - 4\sin^3 x}{\sin x} - \frac{4\cos^3 x - 3\cos x}{\cos x}$
 $= 3 - 4\sin^2 x - 4\cos^2 x + 3$
 $= 6 - 4(\sin^2 x + \cos^2 x)$
 $= 6 - 4$

Ac f est la cte égale à 2 sur D

EX5

1) $i^7 = (i^2)^3 \times i = (-1)^3 \times i = -i$

2) $(1-2i)(-1+3i) = -1 + 2i + 3i - 6i^2 = 5 + 5i$

3) $\frac{1}{i} = \frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{-i^2} = -i$

4) $\frac{1}{1-i} = \frac{1+i}{1+i} \times \frac{1-i}{1-i} = \frac{1+i}{2}$

5) $\frac{2-3i}{1+2i} = \frac{(2-3i)(1-2i)}{5} = \frac{-4-7i}{5}$

6) $(1+i)^3 = 1 + 3i + 3i^2 + i^3 = 1 + 3i - 3 - i = -2 + 2i$

7) $(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)^2 = \frac{1}{2} + \frac{2}{\sqrt{2}\sqrt{2}}i + \frac{i^2}{2} = i$

8) $2(-i+1)(-2i)(-i-1)(-1-i)/(i-1)$
 $= 2 \times -2i \times (1-i) \times -(1+i) \times -(1-i) \times -(1+i)$
 $= -4i \times (1-i)^2 \times (1+i)^2$
 $= -4i \times 16 = -64i$

9) $\overline{(z-1)} = \overline{z-1} = \overline{z} - 1 = -i(\overline{z}-1) = -i(\overline{z}-1)$
 $= -i(\overline{z}-1) = -i\overline{z} + i$

10) $-e^{i\frac{\pi}{8}} = e^{i\pi} \times e^{i\frac{\pi}{8}} = e^{i(\pi+\frac{\pi}{8})} = e^{i\frac{9\pi}{8}}$

11) $e^{i\frac{\pi}{8}} = \cos \frac{\pi}{8} + i \sin \frac{\pi}{8} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

12) $e^{i\frac{7\pi}{2}} = e^{i(2\pi+\frac{3\pi}{2})} = e^{i\frac{3\pi}{2}} = -i$

13) $e^{-\frac{5i\pi}{4}} = e^{i(2\pi-\frac{5\pi}{4})} = e^{i\frac{3\pi}{4}}$
 $= \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} = \frac{-\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}$

14) $e^{-i\frac{\pi}{4}} \times e^{i\frac{\pi}{3}} = e^{i(-\frac{\pi}{4}+\frac{\pi}{3})} = e^{i\frac{\pi}{12}}$

15) $\frac{1}{e^{i\theta}} = e^{-i\theta}$

16) $i e^{i\theta} = e^{i\frac{\pi}{2}} e^{i\theta} = e^{i(\theta+\frac{\pi}{2})}$